Quantum Fluctuations for Gravitational Impulsive Waves

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Abstract

Quantum fluctuations for a massless scalar field in the background metric of spherical impulsive gravitational waves through Minkowski and de Sitter spaces are investigated. It is shown that there exist finite fluctuations for de Sitter space.

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Introduction

The possibility that at the root of the mechanism explaining galaxy formation cosmic strings lie is not still ruled out /[1]. We need further experimental data on the anisotropies in the cosmic microwave radiation, lensing of quasar images, or gravitational radiation stemming from the decay of strings to accept or reject this alternative to inflationary quantum fluctuations /[1,2,3].

However, besides their cosmological implications cosmic strings are also of interest from the point of quantum field theory in curved space-time. Since the space-time generated by a straight cosmic string possesses a conical structure, it gives rise to finite vacuum fluctuations of quantized fields. These effects have been calculated by many authors [4,5]. Another effect is seen when the straight cosmic string splits into two ends. This results in the emission of a spherical wave.

Exact solutions of Einstein's field equations for spherical impulsive waves emitted by snapping cosmic strings was found by Nutku and Penrose /[6]. It is given by the metric

$$ds^2 = \left(2dudv + 2u^2|d\zeta + \frac{v\Theta(v)h(\bar{\zeta}d\bar{\zeta}}{2u}|^2\right), \tag{1}$$

where h is the Svhwarzian derivative of an arbitrary function $f(\zeta)$. ζ is the stereographic coordinate on the sphere, u is a Bondi-type luminosity distance and v is a null coordinate which can be regarded as retarded time.

One may investigate whether the spherical impulsive wave emitted when a cosmic string snaps gives rise to finite vacuum fluctuation phenomena. In Ref. 7 we investigated the vacuum fluctuations in the background metric of special cases of these solutions.

One can construct similar solutions for impulsive gravitational waves propagating through the de Sitter universe /[8]. Here one has to simply multiply the Minkowski solution by a conformal factor $(1+\frac{\Lambda uv}{6})^{-2}$. Although the Nutku- Penrose solution is not conformally related to Minkowski space., an obstruction to the presence of finite vacuum fluctuations may sneak in since we are perturbing about the Minkowski solution. This new solution in de Sitter space has a parameter, the cosmological constant Λ, with the dimensions of mass squared. In thuis case, perturbation theory is performed about a space where conformal

symmetry is explicitly broken. This allows us to investigate if our null results in Ref. 7 persists in the absence of conformal symmetry even in the space about which we perform our perturbation expansion.

To search for vacuum fluctuations, the usual method is the calculation of vacuum expectation value,VEV, of the the stress-energy tensor of a scalar field in the background metric proposed for the spherical wave /[9]. This is achieved by calculating the two point function, $G_F(x,x')$ for the model as a solution of the equation $(\Box+\xi R)G_F=-\frac{\delta(x,x')}{\sqrt{-g}}$ and differentiating the result to obtain $<0|T_{\mu\nu}|0>$ after the coincidence limit is taken. The d'Alembertian, denoted by $\Box$, in the above expression is written in the background metric of Eq.(1). R is the Ricci scalar and ξ is a constant which determines the coupling of the field to the metric.

Here we will first perform the calculation by expanding about the Minkowski solution. This means in calculating VEV we will use the vacuum for the empty Minkowski space-time with no boundaries. In other words we will define our vacuum state in the usual way by choosing positive frequency modes with respect the Killing vector $\frac{\partial}{\partial t}$ of flat Minkowski space-time. Then we will obtain the de Sitter solutions by multiplying this solution by conformal factors. We will obtain the de Sitter solution by multiplying this solution by conformal factors .

For technical reasons we first take a massive scalar field, but take the mass equal to zero at the end of the calculation not to introduce any mass parameters to the Minkowski calculation. We have more control on the consequences of such a parameter when it is introduced via the de Sitter universe construction. We use the conformal coupling of the scalar field to the metric. This is important in the de Sitter case where the Ricci scalar is not zero. In the Minkowski case the Ricci scalar is zero; so, whether we use minimal or conformal coupling does not matter.

In Ref. 7 we studied a special case of the metric given in Eq. (1) and used the form $f=(\zeta)^{1+\delta+i\epsilon}$. The arbitrary function in the solution will be the Schwarzian derivative if this function.

We took only the first order terms and found that h is proportional to δ and ϵ. Since in the Nutku-Penrose solution the impulsive wave is generated in a process where a cosmic

string snaps, we want a small parameter multiplying any possible non zero fluctuation effect to act as a signature of the string. Here δ and ϵ serve this purpose.

In this paper we take a generalized form of this function as $f = \left(\frac{A\zeta+B}{C\zeta+D}\right)^{1+i\sqrt{2}\epsilon}$. We took this form for f , since we knoe that the Schwarzian derivative of this expression is zero when ϵ is zero. h is proportional to ϵ in the first order.

Our previous choice for f is a degenerate form of this expression, where either A and D or C and B are taken to be zero. We take δ zero in the new calculation since we know from our previous work /7 that for fluctuations the case when δ is not equal to zero looks essentially the same as the case when ϵ is not to zero. The singular part of the Greens function is the same, only factors multiplying this term changes; so, we conclude that taking only one of the parameters finite suffices.

If we agree not to take both A and C in the above expression equal to zero, we can reduce the new f to the form $f = \left(\frac{\zeta+\frac{B}{A}}{\zeta+\frac{D}{C}}\right)^{1+i\sqrt{2}\epsilon}$ up to an overall factor in front of this term which can be absorbed in the expansion parameter ϵ. In our specific calculation we take $\frac{B}{A} = -1, \frac{D}{C} = 1$ for convenience. Since we studied the $A = 0$ and the $C = 0$ cases seperately, we think that our new case exhausts all the interesting cases. Whether there are vacuum fluctuations or not should not depend upon the location of the zeroes and poles of f; so, taking the poles at plus and minus one should be able to represent the general behaviour of this set of trial functions.

In the main part of the article, we describe the stress-energy tensor calculation when $f = \left(\frac{\zeta-1}{\zeta+1}\right)^{1+i\sqrt{2}\epsilon}$. In Section III we apply our results to the de Sitter case. Since the de Sitter solution is just conformally related to the Minkowski case, we obtain the de Sitter space Greens functions from the Minkowski ones just by multiplying them by conformal factors. The end results is different, though. In all the cases studied, we find one non-vanishing component of the vacuum expectation value of T_{vv} which is proportional to $\delta(v)\Lambda^2$. The metric we used had only one non identically vanishing component for the curvature tensor which is also proportional to the Dirac delta function of the variable v.The same behaviour is seen in the stress-energy tensor in de Sitter universe.

Calculationg for the Wave Propagating in Minkowski Space

We start with the Nutku-Penrose metric, given in eq.(1) for the case where h is the Schwarzian derivative of $f = \left(\frac{\zeta-1}{\zeta+1}\right)^{1+i\sqrt{2}\epsilon}$ and treat only first order terms in ϵ. Here taking complex values for the exponent corresponds to taking a rotating string, as explained in Reference 6.

In the first order in ϵ,h is given by

$$h = \frac{i8\epsilon}{(x+iy+1)^2(x+iy-1)^2} \tag{2}$$

for $\zeta = \frac{x+iy}{(2)^{1/2}}$. This choice gives us the metric

$$ds^2 = 2dudv - (u^2+\epsilon P_1)dx^2 - (u^2-\epsilon P_1)dy^2 + 2\epsilon P_2 dxdy \tag{3}$$

where

$$P_1 = 64uvxy\frac{(x^2-y^2-1)}{P^2} \tag{4}$$

$$P_2 = 16uv\frac{[(x^2-y^2-1)^2-4x^2y^2]}{P^2} \tag{5}$$

$$P = (x^2-y^2-1)^2+4x^2y^2. \tag{6}$$

Just note that for technical purposes we first do the calculation for the massive case. Here the mass used can be taken as an infrared parameter. We take the zero mass limit before the coincidence limit is taken. We derive the d'Alembertian operator $(-g)^{-1/2}L$, written for the Nutku-Penrose metric. The operator, L , reads, for $v>0$,

$$L = 2u^2\frac{\partial^2}{\partial u\partial v} + 2u\frac{\partial}{\partial v} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + 2m^2u^2 + \frac{\epsilon}{u^2}\left[P_1(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}) - 2P_2\frac{\partial^2}{\partial x\partial y}\right]. \tag{7}$$

We note that the Nutku-Penrose metric does not allow first order derivatives in x and y, for any h used in the original metric in this order.

In order to solve Eq.(7) we use a perturbative scheme. The zeroth order part, which we call L_0, is the flat Minkowski space d'Alembertian function written in our coordinate system. Our vacuum will be that of the Minkowski space with no boundaries.

At this point we may use an expansion of the Feynman Green Function using the integral equation approach. Then

$$G_F = G_F^0 + G_F^0VG_F^0 + ...,$$

where G_F^0 is the free Feynman propagator and V is the perturbing part.

We will, instead, start with the Sturm-Liouville equation $L\phi_\lambda = \lambda\phi_\lambda$ and the explicit construction of the Green function G_F will use the eigenfunctions of this equation. G_F will be given by

$$G_F = -\sum_\lambda \frac{\phi_\lambda(x)\phi_\lambda^*(x')}{\lambda}. \tag{8}$$

Here the eigenfunctions of the Sturm-Liouville problem form a complete set. We perform the calculation to first order in ϵ.

We expand the operator L, the eigenfunction ϕ_λ and the eigenvalue λ, in powers of ϵ,

$$(L_0 + \epsilon L_1)(\phi_0 + \epsilon\phi_1 + \cdots) = (\lambda_0 + \epsilon\lambda_1 + \cdots)(\phi_0 + \epsilon\phi_1 + \cdots). \tag{9}$$

which gives

$$L_0\phi_0 = \lambda_0\phi_0, \tag{10}$$

$$L_1\phi_0 + L_0\phi_1 = \lambda_1\phi_0 + \lambda_0\phi_1 \tag{11}$$

where

$$L_0 = (2u^2\frac{\partial^2}{\partial u\partial v} + 2u\frac{\partial}{\partial v} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}) + 2m^2u^2 \tag{12}$$

$$L_1 = \frac{1}{u^2}\left[P_1(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}) - 2P_2\frac{\partial^2}{\partial x\partial y}\right]. \tag{13}$$

L_0 is the d'Alembertian for the Minkowski case. P_1 and P_2 are as defined in Eq.s (4) and (5).

The zeroth order solutions ϕ_0 and λ_0 are found easily .

$$\phi_0 = \frac{e^{iRv}e^{ik_1x}e^{ik_2y}e^{\frac{-iK}{2Ru}}e^{\frac{im^2u}{R}}}{(2\pi)^2u\sqrt{2|R|}}. \tag{14}$$

Here $\lambda_0 = K - k_1^2 - k_2^2$. k_1, k_2, R and K are constants definig the different modes. Note that ϕ_0 is also a solution in Minkowski space.

We also find $\lambda_1 = (\phi_0, L_1\phi_0) = 0$ due to the particular form of the operator L_1.

To solve the inhomogenous equation for ϕ_1, we take $\phi_1 = \phi_0 f$. Then our eq.(11) reduces into $L_1^{'}f = vH$, explicitly written as

$$\left[2iRu^2\frac{\partial}{\partial u} - 2i(k_1\frac{\partial}{\partial x} + k_2\frac{\partial}{\partial y}) - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + 2u^2\frac{\partial^2}{\partial u\partial v} + \frac{iK}{R}\frac{\partial}{\partial v}\right] f$$

$$= \frac{v}{u}\left[\frac{64xy(x^2-y^2-1)(k_1^2-k_2^2)}{[(x^2-y^2-1)^2+4x^2y^2]^2} - 32\frac{k_1k_2[(x^2-y^2-1)^2-4x^2y^2]}{[(x^2-y^2-1)^2+4x^2y^2]^2}\right]. \tag{15}$$

A particular solution of this equation is of the form

$$f = vF_1(x,y,u) + F_2(x,y,u). \tag{16}$$

We find this ansatz by assuming the solution to be of the form

$$f = G_1(v)F_1(x,y,u) + F_2(x,y,u).$$

When this form is substituted in Eq. (15), the form given in Eq.(16) emerges, with two new coupled differential equations for F_1 and F_2.

$$L_2F_1 = H, \tag{17}$$

$$L_2F_2 = (-2u^2\frac{\partial}{\partial u} - \frac{iK}{R})F_1. \tag{18}$$

Here H is as defined inside the square parenthesis on the righr hand side of eq. (15) and L_2 is given as

$$L_2 = 2iRu^2\frac{\partial}{\partial u} - 2i(k_1\frac{\partial}{\partial x} + k_2\frac{\partial}{\partial y}) - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \tag{19}$$

Since ϕ_1 is the product of ϕ_0 with f, the boundary conditions are dictated by ϕ_0, and the form given by Eq.(16) obeys the boundary conditions fixed by the zeroth order term, which is the solution in empty Minkowski space.

At this point we change the variables and define $s = \frac{1}{u}$. We use both variables in our expressions. We also note that if we use the new variables $z = x+iy, \overline{z} = x-iy$, both the inhomogenous term H and the operator itself seperate. We find that we get

$$\left[-2iR\frac{\partial}{\partial s} - 2i\left((k_1+ik_2)\frac{\partial}{\partial z} + (k_1-ik_2)\frac{\partial}{\partial \overline{z}}\right) - 4\frac{\partial^2}{\partial z\partial\overline{z}}\right]F_1 =$$

$$-8is\left[\frac{(k_1-ik_2)^2}{(\overline{z}-1)^2(\overline{z}+1)^2} - \frac{(k_1+ik_2)^2}{(z-1)^2(z+1)^2}\right]. \tag{20}$$

and a similar equation for F_2.

Upon solving the new differential equations, we find particular solutions

$$F_1 = s\big[2ik_1\big(\tan^{-1}\frac{2y}{x^2+y^2-1} - \big(\frac{y}{(x-1)^2+y^2} + \frac{y}{(x+1)^2+y^2}\big)\big)$$
$$-ik_2\big(\log\frac{(x+1)^2+y^2}{(x-1)^2+y^2} - \frac{2(x+1)}{(x+1)^2+y^2} - \frac{2(x-1)}{(x-1)^2+y^2}\big)\big] \tag{21}$$
$$-R\big(x\tan^{-1}\frac{2y}{(x^2+y^2-1)} + \frac{y}{2}\log\frac{(x-1)^2+y^2}{(x+1)^2+y^2}\big)$$

and

$$F_2 = \left(\frac{iKs}{2R} - 1\right) M_1(x,y) + \frac{iK}{2(k_1^2+k_2^2)}(k_1 M_2(x,y) - k_2 M_3(x,y)), \tag{22}$$

where

$$M_1(x,y) = -2y\log\left(\frac{(x+1)^2+y^2}{(x-1)^2+y^2}\right) - 2x\tan^{-1}\frac{y}{x+1} + 2x\tan^{-1}\frac{y}{x-1} \tag{23}$$

$$M_2(x,y) = 4xy\log\left(\frac{(x+1)^2+y^2}{(x-1)^2+y^2}\right) + 2(x^2-y^2-1)(\tan^{-1}\frac{y}{x+1} - \tan^{-1}\frac{y}{x-1}) + 4y \tag{24}$$

$$M_3(x,y) = (2x^2-2y^2-2)\log\left(\frac{(x+1)^2+y^2}{(x-1)^2+y^2}\right) - 4xy(\tan^{-1}\frac{y}{x+1} - \tan^{-1}\frac{y}{x-1}) + 4x \tag{25}$$

Now these solutions are used to form the Greens function, G_F, of the original operator given in eq.(7). Since the zeroth order part of G_F is the same as the flat space Greens function, we calculate only the first order part.

To obtain G_F, in first order, we take

$$G_F = \frac{i\epsilon}{2}\int_{-\infty}^{\infty} dk_1 \int_{-\infty}^{\infty} dk_2 \int_{-\infty}^{\infty} dK \int_{-\infty}^{\infty} \frac{dR}{|R|}\frac{e^{iR(v-v')}}{(2\pi)^4 uu'} e^{ik_1(x-x')} e^{ik_2(y-y')} e^{\frac{-iK}{2R}\left(\frac{1}{u}-\frac{1}{u'}\right)}$$
$$\times e^{im^2\frac{(u-u')}{R}} \frac{\big(\Theta(v)f(x,y,u,v) + \Theta(v')f^*(x',y'u',v')\big)}{K-k_1^2-k_2^2}. \tag{26}$$

with $f(x,y,u,v)$ defined in Eq. (16). Here we use the Schwinger prescription to regularize the denominator and use the formula $\frac{1}{A-i\delta} = i\int_0^\infty e^{-i\alpha A-\delta\alpha}d\alpha$ where δ is a positive constant approaching zero. The infinite integration over k_1, k_2 is regularized by the denominator. The result is the Feynman propagator with its usual contour.

We need G_F to calculate the vacuum expectation value of the stress-energy tensor $T_{\mu\nu}$. We are particularly interested in whether $< T_{vv} >$ has a finite part. Since in T_{vv} x or y derivatives do not exist, we take the coincidence limit in x and y at this stage.

This puts terms of the type $(x-x')^2$ and $(y-y')^2$ to zero. Such terms either multiply the expressions obtained, or appear in the 'geodesic distance' term $\sigma=(u-u')(v-v')-\frac{uu'}{2}((x-x')^2+(y-y')^2)$, which appears in the denominator. Taking the coincidence limit in x and y only simplifies the calculation at this stage and does not affect the end result.

When we take x equals x' and y equals y', all the terms that are odd in k_1 and k_2 vanish. The remaining terms are given by

$$G_F(s,s',v,v',x,y)=\frac{-i\epsilon}{uu'2(2\pi)^4}\int\frac{dR}{2|R|}\int dK\int dk_1\int dk_2\int_0^{\infty}d\alpha e^{\frac{-iK}{2R}\left(\frac{1}{u}-\frac{1}{u'}\right)}$$

$$\times e^{iR(v-v')}e^{im^2\frac{(u-u')}{R}}e^{-i\alpha\left(K-k_1^2-k_2^2\right)-\alpha\delta}$$

$$\times\Big[\left(\frac{K}{2R}(s\Theta(v)-s'\Theta(v'))+i(\Theta(v)+\Theta(v'))\right)M_1(x,y)+2RM_4(x,y)(v\Theta(v)-v'\Theta(v'))\Big] \tag{28}$$

where

$$M_4(x,y)=\frac{1}{2}y\log\frac{(x+1)^2+y^2}{(x-1)^2+y^2}-x\tan^{-1}\frac{2y}{x^2+y^2-1}, \tag{29}$$

and $M_1(x,y)$ was defined in Eq. (23).

We perform the integrations over k_1,k_2,K and R. The result of the α integration can be shown to result in Hankel functions which degenerates into a monomial when the m going to zero limit is taken. Hence for the massless field case the Greens Function G_F reads

$$G_F=\left(\frac{\epsilon}{16\pi}\right)\Big[\frac{1}{(u-u')(v-v')}\Big(\left(\frac{(u\Theta(v')-u'\Theta(v))}{(u-u')}-(\Theta(v)+\Theta(v'))\right)M_1(x,y)$$

$$+M_4(x,y)\frac{(v\Theta(v)-v'\Theta(v'))}{(v-v')}\Big)\Big] \tag{30}$$

We note that the singularity structure in the coincidence limit of this expression displays exactly that of the free Minkowski Green Function. There is just a modulating factor which is finite in the coincidence limit. Since our perturbative solution ϕ_1 is given as a factor times the Minkowski solution, the singularity structure of the Minkowski vacuum is carried over this solution. This part may be regularized by renormalizing the free parameters of the theory.

The empty space Green function goes as $\frac{1}{4\pi^2\sigma}$. Here σ is the square of the geodesic distance between the two points given by Eq. (27). A well known theorem /[10] states that if the metric is of the Minkowskian shape for a certain region of space-time, the Green Function will be of the Hadamard type /[11],

$$G_F = \frac{A}{\sigma} + Blog\sigma + C,$$

where A,B,C are finite quantities at the coincidence limit; σ is the geodesic distance between the points of G_F. Our hope, in this calculation, was to find a finite term C given in the above formula. Unfortunately our end had B and C identically zero. We choose our boundary conditions of the renormalized Green function of the renormalized Greens function so that we get zero as u and v go to infinity. This condition fixes C_0 in the expansion $C = \Sigma C_l \sigma_l$, therefore completely fix C uniquely equal to zero.

Another way of obtaining this result will be applying the Adler-Lieberman-Ng prescription /[12] to this problem. An ambiguity may arise only by a local conserved tensor which comes out to be proportional to a_2 of the adiabatic expansion in the DeWitt-Schwinger method /[13]. This term is identically zero in our case; so, even this possible ambiguity is not present. Other references to the uniqueness property of the VEV of the stress-energy tensor can be found in Ref. 14.

Calculation for the Wave Propagating in de Sitter Universe

We conjecture that we have to introduce a mass parameter to our space in order to get non zero result for the vacuum fluctuations if we insist performing our calculation perturbatively. One can check whether this is true by doing the similar calculation in de Sitter universe. Indeed we find that our end result changes if we look at the spherical impulsive gravitational waves propagating through the de Sitter universe. It is shown that /[8] an impulsive spherical gravitational wave solution exists in de Sitter space. Here one has to simply multiply the Minkowski solution by a conformal factor $(1 + \frac{\Lambda uv}{6})$ where Λ is the cosmological constant which comes out proportional to Ricci scalar in this metric. The metric now reads

$$ds^2 = (1 + \frac{\Lambda uv}{6})^2 \left(2dudv + 2u^2 |d\zeta + \frac{v\Theta(v)h(\bar{\zeta})d\bar{\zeta}}{2u}|^2 \right) \qquad (31)$$

We take the same h as before, as given in eq.(2). Since we use conformal coupling, the operator L is modified to include also the Ricci scalar term for this metric which is no longer zero.

We know from general arguments that [/9]

$$G_F^S = (1 + \frac{\Lambda uv}{6}) G_F^M(x, x')(1 + \frac{\Lambda u'v'}{6}) \tag{32}$$

where G_F^S and G_F^M are the de Sitter and Minkowski space Greens functions respectively.

Note that

$$(1 + \frac{\Lambda uv}{6})(1 + \frac{\Lambda u'v'}{6}) = (1 + \frac{\Lambda UV}{6})^2 + \frac{\Lambda}{12}(u - u')(v - v')$$
$$+(\frac{\Lambda}{12})^2 \left(-(u - u')^2 V^2 - (v - v')^2 U^2 + \frac{(u - u')^2 (v - v')^2}{4} \right) \tag{33}$$

where $U = \frac{u+u'}{2}, V = \frac{v+v'}{2}$. Using this expansion we see that for G_F^S we get terms in the coincidence limit that give finite terms to $< T_{vv} >$

$$\lim_{u \to u', v \to v'} \frac{\partial^2}{\partial v \partial v'} \left[\frac{\Lambda^2 (u - u')^2 (v - v')^2}{(24)^2} G_F^M(x, x') \right]$$
$$= \frac{-u\epsilon\Lambda^2}{(96)^2\pi} \left(2x \tan^{-1} \frac{y}{x+1} - 2x \tan^{-1} \frac{y}{x-1} - 2y \log \left(\frac{(x-1)^2 + y^2}{(x+1)^2 + y^2} \right) \right) \delta(v) \tag{34}$$

We find that in first order calculation for spherical impulsive waves propagating through the de Sitter universe, we get one non-zero component of the stress-energy tensor.

If we perform the similar calculation for the cases studied in Ref.7, we get nonzero contribution, given in the same way. Here h in eq.(1) comes from $f = (\zeta)^{1+\delta+i\epsilon}$. Using the results given in Ref.7, we get

$$T_{vv} = \frac{-\Lambda^2 u \delta(v)}{2(96)^2\pi} \left(\delta \log(x^2 + y^2) - 4\epsilon \tan^{-1} \frac{y}{x} \right). \tag{35}$$

This result is finite, contrary to the case for propagating in Minkowski space.

Conclusion

Here we calculated the stress-energy tensor of a scalar field in the space-time of a spherical impulsive gravitational wave propagating through the Minkowski and de Sitter universes.

We verified the common knowledge that fluctuations which are null in Minkowski space may become finite in de Sitter space. We found that VEV of one component of the energy momentum tensor in first order is proportional to $\delta(v)$ which is the signature of the impulsive wave solution, in de Sitter space. This is the same factor to which the only nonzero component of the curvature tensor is also proportional.

Our results are found in first order perturbation theory, but are proportional to the square of the curvature scalar. They are an example of extracting non trivial information out of first order perturbation theory.

As a technical remark note that first order solutions to the Sturm Liouville problem arising from this metric can be expressed in terms of sum of holomorphic and antiholomorphic functions. This occurs after we factorize the solution into two parts, one part proportional to the flat space solution. This fact simplifies our calculation, since then second order differential equation decomposes in to a pair of first order equations which are easily integrated. In our previous papers /7, we used brute force Greens function method to integrate the inhomogenous equation. Since we have to perform integrals where the integration range is infinite, there were points where regularizations were used. In the new method we do not need any regularizations in calculating G_F. The result is free from any possible ambigiuties the regularization may cause.

We reduce first order eigenfunction calculation to the solution of a couple of first order differential equations. This is possible since both the inhomogenous term and the differential operator decomposes into a sum of two terms if a set of variables are used.

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REFERENCES

1. A.Vilenkin and E.P.S. Shellard, *Cosmic Strings and other Topological Defects*, Cambridge University Press, Cambridge (1994);
   M.B. Hindmarsh and T.W.B. Kibble, Reports on Progress in Physics, 58 (1995) 477.
2. L. Perivolaropoulos, " Cosmic String Theory: The Current Status", MIT preprint, MIT-CTP-2375 (1994).
3. Ed. by G.W. Gibbons, S.W. Hawking and T. Vachaspati, *The Formation and Evolution of Cosmic Strings*, Cambridge University Press, Cambridge (1990).
4. T. Vachaspati and A. Vilenkin , Phys. Rev. D31 (1985) 3052.
5. L. Parker , Phys. Rev. Lett. 59 (1987) 1365; A. C. Smith, in reference 3, p.263; V. Sahni, Modern Phys. Lett. A 3 (1988) 1425; T.M. Helliwell and K.A. Konkowski, Phys. Rev. D34 (1986) 1908; B. Linet, Phys. Rev. D33 (1986), D35 (1987) 536.
6. Y. Nutku and R. Penrose, Twistor Newsletter 34 (1992) 9.
7. M.Hortaçsu, J. Math. Phys., 34 (1993) 690; M. Hortaçsu and R. Kaya, J. Math. Phys., 35 (1994) 3043.
8. P.A. Hogan, Phys. Lett. A171 (1992) 21.
9. N.D. Birrell and P.C.W. Davies, *Quantum Fields in Curved Space*, Cambridge University Press, Cambridge (1982).
10. S.A.Fulling, M. Sweeny, R.M.Wald, Comm. Math. Phys. 63 (1978) 257.
11. J. Hadamard, *Lectures on Cauchy's problem in linear partial differential equations* (Yale University Press,1923).
12. S. L. Adler, J. Lieberman, Y. J. Ng, Ann. Phys (N.Y.) 106 (1977) 279; S. L. Adler, J. Lieberman, Ann. Phys. 113 (1978) 294; R. M. Wald, Phys. Rev. D17 (1978) 1477.
13. J.Schwinger, Phys. Rev. 82 (1951) 664; B.S. DeWitt, *The dynamical Theory of Groups and Fields*,(Gordon and Breach, New York, 1965); B.S.DeWitt, Phys. Reports 19C (1975) 297.
14. R.M.Wald, Comm. Math. Phys. 54 (1977)1; Stephen A. Fulling, *Aspects of Quantum Field Theory in Curved Space-Time*, (Cambridge University Press, Cambridge 1989), especially Chapter 9 on Renormalization; a summary is found in Chapters 6 and 7 of reference 9.